

Another Length Scale in String Theory?

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We suggest that some of the remarkable results on stringy dynamics which have been found recently indicate the existence of another dynamical length scale in string theory that, at weak coupling, is much shorter than the string scale. This additional scale corresponds to a mass $\sim m_s/g_s$ where m_s is the square root of the string tension and g_s is the string coupling constant. In four dimensions this coincides with the Planck mass.

1. Introduction

Our understanding of the dynamics of string theory has dramatically increased recently [1-37]. We now understand, via strong coupling-weak coupling dualities, that apparently different theories are different regions of the same theory. We also understand that apparently disjoint vacuum states are in fact connected.

The existence of certain kinds of solitons is crucial to these dynamical insights. At weak string coupling, g_s^2 , these solitons generically have masses $m_{\text{sol}} \sim m_s/g_s^2$, or for those carrying Ramond-Ramond (RR) charge, $m_{\text{sol}} \sim m_s/g_s$ [12]. Here m_s is the basic string mass scale $m_s \sim \sqrt{T}$ where T is the string tension. When the coupling is weak these solitons are much heavier than m_s .

This by itself is not surprising. In field theory, even though solitons define a new heavy mass scale, this scale does not have dynamic consequences at short distance or high momentum. The basic reason for this is that solitons, while heavy, are also big. As an example consider a nonabelian gauge theory spontaneously broken to $U(1)$. There are magnetic monopoles in this theory whose mass at weak coupling, g^2 , is $m_{\text{mon}} \sim m_W/g^2$. Here m_W is the mass of a typical massive gauge vector boson. Even though $m_{\text{mon}} \gg m_W$ we do not expect the short distance behavior of this gauge theory to be anything but that of a theory of weakly interacting gauge bosons. $1/m_W$ is the shortest dynamical length scale in the problem. It is also the semiclassical “size” of the monopole, far larger than $1/m_{\text{mon}}$.

In string theory we have come to believe that the shortest dynamical length scale in the theory is $1/m_s$. There are many indications of this, including gaussian fall-off of high momentum fixed angle scattering, Regge behavior, $R \rightarrow 1/R$ duality, and the Hagedorn transition. Because of the field theory intuition just mentioned the existence of heavy solitons has not caused us to question this conventional wisdom.

The aim of this paper is to suggest that, on the contrary, some of the new information on string dynamics seems to indicate that m_s/g_s serves as another *dynamical* scale in string theory. The evidence we will present for this additional scale is quite indirect. It may be that there is an explanation for it that does not require the existence of another scale. Nonetheless the evidence is sufficiently puzzling that it seems worthwhile to present it here.

This is not the first suggestion of another dynamical scale in string theory. Susskind has argued [38] that at energies above the Planck mass m_P ($\sim m_s/g_s$ in four dimensions)

strings must become black holes and behave rather differently than perturbative strings. He points to the flattening of the gaussian falloff of fixed angle scattering as the order of perturbation theory is increased [39] as an indication of a possible change in high momentum behavior.

2. Logarithms

The evidence for another length scale comes from new exact results about certain long distance quantities in low energy effective Lagrangians. Long distance quantities can give some information about short distance behavior because of the high momenta present in loops of virtual particles that contribute to them.

To see how this works let us examine a field theoretic example. A large number of exact low energy results in supersymmetric field theory are now available due to the enormous recent progress in this subject spearheaded by Seiberg [40].

To be explicit we will consider the exact low energy effective action of $\mathcal{N} = 2$ $SU(2)$ supersymmetric gauge theory determined in the beautiful work of Seiberg and Witten [41]. The low energy effective dual $U(1)$ coupling constant $\tau_D = 4\pi i/e_D^2 + \theta/2\pi$ depends on the modulus of the theory, the complex valued gauge invariant vacuum expectation value of the adjoint scalar field. Let $Z = 0$ be the point of this moduli space where monopoles become massless. Near this point

$$\tau_D(Z) \sim \frac{i}{2\pi} \log(Z) . \quad (2.1)$$

This singular behavior is explained in [41] as the infrared divergent charge screening due to a light magnetic monopole. Such an explanation requires interpreting the logarithm in (2.1) as $-\log(m_{uv}/m_{mon})$ where m_{mon} is the mass of the magnetic monopole in the theory and m_{uv} is the ultraviolet cutoff in the loop integral. We might think of m_{uv} as the inverse “size” of the monopole. It represents the scale at which a description purely in terms of light pointlike monopoles breaks down and the logarithmic functional form in (2.1) ceases to be accurate. The size of m_{uv} is a piece of knowledge about short distance physics gained from long distance measurements.

In theories with extended supersymmetry, states in reduced multiplets obey a BPS mass formula. For the theory in [41] as $Z \rightarrow 0$ this formula give $m_{mon} \sim |Z|\Lambda$ and $m_W \sim \Lambda$ where Λ is the asymptotic freedom mass scale. Using these relations we see that $m_{uv} \sim m_W$ up to factors of order one, in accord with semiclassical intuition about

the “size” of a monopole. This interpretation is bolstered by the analysis of the $SU(N)$ generalization of the Seiberg-Witten solution [43] given in [42]. In the $SU(N)$ case there are $N - 1$ $U(1)$ gauge fields at low energy. At large N there is a large hierarchy of different length scales in the theory at the massless monopole point. The W masses there range from $\sim \Lambda$ to $\frac{1}{N^2}\Lambda$. For each $U(1)$ factor (indexed by a) the analog of (2.1) ceases to be a good description when m_{mon_a} becomes $\sim m_{\text{uv}_a} \sim m_{W_a}$ where m_{W_a} is the mass of the lightest charged vector boson that couples to $U(1)_a$. This all seems very sensible.

Let us now turn to string theory and in particular examine Strominger’s remarkable resolution of the conifold singularity [15]. He studies 4D type IIB superstring theory compactified on a Calabi-Yau manifold (CY) that develops a conifold singularity as a modulus denoted by Z goes to zero. This theory has $\mathcal{N} = 2$ 4D supersymmetry and Z is the scalar component of a vector multiplet. The $U(1)$ vector gauge field in this multiplet is a RR field. Its effective low energy coupling constant (related by supersymmetry to the geometry of moduli space) has been computed exactly at string tree level using mirror symmetry [45]. This result should be exact in quantum string theory [15] because the IIB dilaton lies in a neutral hypermultiplet which cannot couple to the vector multiplets by $\mathcal{N} = 2$ supersymmetry. The result for the effective coupling τ_{RR} as $Z \rightarrow 0$ is precisely (2.1):

$$\tau_{RR}(Z) \sim \frac{i}{2\pi} \log(Z) . \quad (2.2)$$

Strominger explains this singularity¹ as the screening due to an (electrically) charged light gravitational soliton referred to as a “black hole.” Again this explanation requires interpreting the logarithm in (2.2) as $-\log(m_{\text{uv}}/m_{\text{bh}})$ where m_{bh} is the mass of the black hole and m_{uv} is the ultraviolet cutoff in the loop integral. The BPS formula for this RR charged state gives, for $Z \rightarrow 0$,

$$m_{\text{bh}} \sim |Z|m_s/g_s . \quad (2.3)$$

This implies that here $m_{\text{uv}} \sim m_s/g_s$! If m_{uv} were $\sim m_s$ then τ_{RR} would be $\sim \log(Z/g_s)$ implying a coupling between RR vector multiplets and the neutral dilaton hypermultiplet D , ($g_s \sim e^D$), and in particular a nonvanishing three point vertex between two RR vector bosons and a zero momentum dilaton. Such couplings are forbidden by $\mathcal{N} = 2$ supersymmetry [44] [15].

¹ In the limit $m_{\text{bh}} \ll m_s$.

Somehow this black hole seems to be behaving like a pointlike four dimensional particle down to length scales many times smaller (for small g_s) than either string or compactification lengths. This does not seem very sensible.

Even before we address string scales there is a problem. At some compactification mass scale m_c (which could be much less than m_s) the theory stops looking like a four dimensional theory and starts looking ten dimensional. Why do we keep doing a four dimensional loop momentum integral? One possible explanation for this ² is that the black hole soliton is “locked” in position on the six dimensional CY.

Perturbative orbifold compactified string theory displays an analogous locking phenomenon. Consider an orbifold compactification from ten to four dimensions where the radius of the orbifold is $R = 1/m_c$ and assume $m_c \ll m_s$. In the untwisted sector there are massless states and Kaluza-Klein (KK) excitations above them of mass $\sim m_c$ (as well as stringy excitations). These KK states cause the theory to look ten dimensional at energies above m_c . In the twisted sector, though, there are no KK excitations. In world sheet language, there are no zero modes of the X fields and the only excitations come from oscillators. In spacetime language, any attempt to move the string away from the orbifold singularity on which it is trapped changes its length and hence costs string tension scale energy.

Strominger [15] argues that the black hole should be thought of as a threebrane wrapped around the collapsing three-cycle that defines the conifold singularity. It is plausible that any excitation of this wrapped threebrane away from the three-cycle will change the world volume and hence cost brane tension energy and be very massive. This would be locking.

Perhaps there is an indirect (and partially circular!) argument for locking along the following lines. If the black hole were not locked its ability to move in the six compact dimensions would be represented by charged KK modes in the 4D field theory whose mass was $\sim m_c$. But m_c depends on Kahler moduli which are part of neutral $\mathcal{N} = 2$ hypermultiplets. Integrating out such fields³ should give Kahler moduli dependence to τ which is forbidden by $\mathcal{N} = 2$.

² This explanation was suggested to me by Tom Banks and Andy Strominger. My understanding of it was shaped by conversations with them and with Jeff Harvey, Emil Martinec and Greg Moore.

³ Which should, as Emil Martinec pointed out, not in general be part of reduced $\mathcal{N} = 2$ multiplets. Andy Strominger noted that multiplets consisting of paired vector and hypermultiplets give zero contribution to τ_{RR} . Perhaps we need to consider higher spin multiplets.

Locking might explain why m_{uv} is much larger than mc . The fact that m_{uv} is much larger than m_s seems a much deeper mystery. How can the theory still display pointlike particle behavior at distances much shorter than the string scale? Why doesn't the theory dissolve into soft mushy strings? We will face this question repeatedly in this paper. One hint comes from an analogy to the Seiberg-Witten case. There m_{uv} was determined by the mass of a field carrying $U(1)$ charge. There are no perturbative string states carrying RR charge, only solitons.⁴ The lightest such state, aside from the light black hole, has mass $\sim m_{\text{uv}}$.

3. Four to Three Dimensions

At first glance there seems to be a serious obstacle to interpreting the conifold logarithm as charge renormalization due to a light black hole. This logarithm is computed in tree level conformal field theory, whose target manifold factorizes into $\mathcal{M} \times \mathbb{R}^4$ where \mathcal{M} is the CY manifold. The logarithm comes entirely from the \mathcal{M} factor which is independent of \mathbb{R}^4 . In particular if we replace \mathbb{R}^4 with $\mathbb{R}^3 \times S^1$ where the radius of the S^1 is R we find an unmodified logarithm. But if the logarithm is reflecting a four dimensional infrared divergence how can it not be sensitive to compactifying one of the four dimensions? At length scales large compared to R the infrared behavior should be three dimensional, not four! But R does not appear in the effective coupling.

This puzzle also seems soluble if we assume that $m_{\text{uv}} \sim m_s/g_s$.⁵ To see how this might work, consider the following model loop integral⁶ for the effective coupling in four dimensions, e_{4D}^2 .

$$1/e_{4D}^2 \sim \int d^4p \frac{1}{(p^2 + m_{\text{bh}}^2)^2} \frac{m_{\text{uv}}^2}{(p^2 + m_{\text{uv}}^2)} \sim \log(m_{\text{uv}}/m_{\text{bh}}) . \quad (3.1)$$

The second factor in the integrand ensures a smooth high momentum cutoff at m_{uv} . Its detailed form is unimportant. In $\mathbb{R}^3 \times S^1$ the model becomes

⁴ Tom Banks first stressed to me that this may be related to the lack of a string scale cutoff.

⁵ This observation was motivated by Ed Witten's remark that in Einstein frame the radius of the compact dimension is R/g_s and hence goes to infinity as g_s goes to zero. So the effects of compactification should disappear at weak coupling.

⁶ This calculation was developed in a discussion with Ronen Plesser.

$$1/e_{3D}^2 \sim \frac{1}{R} \sum_n \int d^3p \frac{1}{(p^2 + (\frac{2\pi n}{R})^2 + m_{\text{bh}}^2)^2} \frac{m_{\text{uv}}^2}{(p^2 + (\frac{2\pi n}{R})^2 + m_{\text{uv}}^2)} . \quad (3.2)$$

If $1/R \ll m_{\text{bh}}$ then we have, schematically,

$$1/e_{3D}^2 \sim 1/e_{4D}^2 + e^{-m_{\text{bh}}R} + e^{-2m_{\text{bh}}R} + \dots + e^{-m_{\text{uv}}R} + e^{-2m_{\text{uv}}R} + \dots . \quad (3.3)$$

This follows from interpreting large R as low temperature or, more formally, by Poisson resummation of (3.2). We now look at the finite R corrections in (3.3). Using (2.3) we see that the term $\exp(-m_{\text{bh}}R)$ becomes $\exp(-\frac{|Z|m_s R}{g_s})$. For Z and $m_s R$ fixed this term is nonperturbative in g_s . In fact it is an example of the mechanism suggested in [12] for producing the nonperturbative effects of “stringy” strength argued to exist in general in [46] on the basis of the large order behavior of string perturbation theory. It is also an example of the instanton effect discussed in [34]. The instanton here is the black hole circling around the S^1 .

Because this R dependence is nonperturbative in g_s it does not contradict the factorization noted above at tree level. The distinction between vector and hypermultiplets disappears in three dimensions so we expect nonperturbative corrections to τ_{RR} [34].

Now consider the term $\exp(-m_{\text{uv}}R)$. For $m_{\text{uv}} \sim m_s$ this term has tree level coupling dependence. This apparently does contradict factorization. But if $m_{\text{uv}} \sim m_s/g_s$ then this term is $\sim \exp(-\frac{m_s R}{g_s})$. This has nonperturbative g_s dependence so again there is no problem. In fact the term is then of the same form as that due to other instantons discussed in [34] corresponding to heavy solitons circling the S^1 , making more plausible the identification of m_{uv} with a heavy soliton mass.

Compatibility with factorization does not really require that $m_{\text{uv}} \sim m_s/g_s$. A weaker g_s dependence, e.g., $m_{\text{uv}} \sim m_s/\sqrt{g_s}$ would suffice. This would correspond to stronger nonperturbative effects $\sim \exp(-\frac{1}{\sqrt{g_s}})$ and hence a large order behavior faster than $(2g)!$ at genus g , which would be surprising.

4. Dual Picture⁷

In the last year or so, strong evidence has accumulated for the validity of various exact string-string dualities [1-37]. We can ask whether these dualities shed some light on the

⁷ The observations in this section were motivated by Ashoke Sen’s and Cumrun Vafa’s remark that the scale m_s/g_s is the natural one in the dual heterotic description of the conifold.

length scale question we have raised. Kachru and Vafa [20] proposed duals for 4D $\mathcal{N} = 2$ type II strings compactified on certain CY manifolds having loci of conifold singularities in their moduli spaces.

One of the duals they found was a 4D $\mathcal{N} = 2$ heterotic string compactified in a such a way that at tree level there is a point of enhanced $SU(2)$ gauge symmetry in its moduli space. The flat space low energy effective field theory near this point is just $SU(2)$ $\mathcal{N} = 2$ gauge theory, the model analyzed by Seiberg and Witten[41]. Seiberg and Witten have taught us that quantum mechanically the $SU(2)$ point splits into two singular points where magnetic monopoles or dyons become massless. The dual image of these points on the type II side are conifold singularities!⁸ The type II dual image of a heterotic magnetic monopole becoming massless is an electrically charged black hole becoming massless at a conifold point. The accompanying figure gives a description of the spectrum in both the heterotic and type II languages in the $Z \ll g_s^{\text{II}} \ll 1$ regime.

By using the magic of second quantized mirror symmetry [21][20] the stringy analog of (2.1) on the heterotic side can be computed from nonperturbatively exact tree level results on the type II side. Assuming that the physics on the heterotic side is essentially that of [41] we can determine m_{uv} . As discussed in section 2 it should just be m_W at the massless monopole point. For flat space physics to be accurate m_W should be much less than the heterotic string scale m_s^{het} , but only by a fixed ratio, independent, e.g., of the value of g_s^{II} on the type II side [36]. But m_W carries RR charge, so its type II image is a heavy nonperturbative (magnetically) charged soliton of mass $\sim m_s^{\text{II}}/g_s^{\text{II}}$. This corresponds to the guess about the identity of m_{uv} made in the previous two sections. The mystery from the heterotic point of view concerns the heterotic images of the neutral perturbative type II states. These states have mass $\sim m_s^{\text{II}} \sim g_s^{\text{II}} m_s^{\text{het}}$ and so are much lighter than the perturbative heterotic states.⁹ Why don't they serve to cut off the logarithm? The exact answer shows no dependence on their mass, since the answer only depends on heterotic vector multiplets. Again, part of the explanation may be that they are neutral.

⁸ There are two such loci in this example, as required.

⁹ The heterotic theory is strongly coupled so this is not implausible, and of course is required by duality.

_____ soliton

_____ perturbative string state

_____ magnetic soliton

_____ W (electric perturbative string state)

_____ perturbative string states (neutral)

_____ light solitons (neutral)

_____ light electric black hole

_____ light magnetic monopole

II

heterotic

5. Finite Momentum¹⁰

So far the signatures we have discussed for physics at scales $\sim m_s/g_s$ are quite indirect. They occur in low energy quantities and all depend in one way or another on the $\mathcal{N} = 2$ supersymmetric decoupling of vector and neutral hypermultiplets. If such a scale is important in string dynamics we would expect to see direct evidence for it in high momentum scattering. To take an extreme example, charged black hole–charged black hole

¹⁰ Much of this section was developed in various discussions with Misha Bershadsky, Mike Douglas, Daniel Friedan, Hiroshi Ooguri, Andy Strominger and Cumrun Vafa.

scattering should show some feature at momenta $\sim m_s/g_s$.

We are a long way from being able to calculate a quantity like this. But we can find some hints about how other momentum scales besides m_s can enter into results. Consider the scattering of two RR $U(1)$ vector bosons near a conifold degeneration. The effective low energy physics should contain a light black hole field, and so we should see a threshold for black hole pair creation. There should be a feature like $(p^2 + m_{\text{bh}}^2)^\alpha$ in the S-matrix. Expanding such a term in powers of p^2 gives a series that looks schematically like

$$S(p) \sim \sum_l (p^2/m_{\text{bh}}^2)^l. \quad (5.1)$$

Using (2.3) we can rewrite this as

$$S(p) \sim \sum_l (p^2 g_s^2/Z^2)^l. \quad (5.2)$$

So a possible signature of this threshold would be a $(p^2/Z^2)^l$ behavior at genus $l+1$ in string perturbation theory as $Z \rightarrow 0$. We also expect terms like $(p^2/m_{\text{bh}}^2)^l (m_s^2/m_{\text{bh}}^2)^n \sim p^{2l} (g_s^2/Z^2)^{n+l}$. These will be quantitatively important since the simple black hole picture will be valid when $m_{\text{bh}} < m_s$. The important point is that no inverse powers of Z appear without accompanying g_s 's.

We can identify a potential source of such terms. Consider the four Z moduli scattering amplitude A_{ZZZZ} (related by supersymmetry to the four vector amplitude) at tree level. This amplitude is related to the curvature of the metric on moduli space defined by the effective $U(1)$ gauge couplings, including τ_{RR} . Differentiating (2.2) twice we find

$$A_{ZZZZ}(p) \sim p^2 g_s^2/Z^2. \quad (5.3)$$

Repeated iterations of this tree level four moduli scattering could produce terms of the type found in (5.2). Because the loop momenta in such a diagram are integrated over, a variety of different terms with different powers of momenta as mentioned above could result.

The above arguments are very similar to scaling arguments presented in [47]. In [47] the authors study the coefficients C_l of certain terms in the effective action of the form $F^{2l} R^2$ where F is the RR field strength and R is the Riemann tensor. They showed that such a term only gets a contribution from genus $l+1$ in string perturbation theory and that in the conifold limit $Z \rightarrow 0$, $C_l \sim (g_s^2/Z^2)^l$ with scaling like the terms in (5.2). The

higher powers of momentum in (5.2) are replaced by nonrenormalizable operators with many derivatives and many RR gauge fields.

Ghoshal and Vafa [24] made further progress by connecting the conifold with a Kazama-Suzuki $SL(2)/U(1)$ coset. This latter, after topological twisting, had already been shown to be equivalent to the $c = 1$ noncritical string by Mukhi and Vafa [48]. The result of this series of mappings is the fascinating result that the C_l are precisely the coefficients of the $c = 1$ matrix model partition function at the self dual radius! ¹¹

The work of Witten [32] motivates a possible physical picture for this occurrence. Witten has argued that in the conifold limit (and in other related degenerations) the quantum corrected geometry of the target manifold develops a long tube whose length goes to infinity as $Z \rightarrow 0$. The dilaton varies linearly along the tube. Such linear dilaton backgrounds are characteristic of noncritical strings. Conjecturally the tube, after topological twisting, will be the $c = 1$ background.¹² The “double scaling limit” of the theory, $g_s \rightarrow 0$, $Z \rightarrow 0$, $\mu \equiv g_s/Z$ held fixed, extracts the physics of the tube. Here μ is to be identified with the continuum coupling (also denoted by μ) of the $c = 1$ matrix model. It is very interesting to observe that μ is just the mass of the black hole in string units, $\mu = m_{\text{bh}}/m_s$. The double scaling limit of the conifold holds the mass of the black hole fixed while the string coupling is taken to zero. It may well be that this limit provides a simpler model in which to study certain aspects of the physics of these light black holes.

Here we content ourselves with the following vague observations about higher energy scales. The $c = 1$ model has nonstandard large momentum behavior. At genus g “tachyon” scattering amplitudes grows like $(k^2/\mu)^{2g}$ at large momentum k [49]. Here k is the momentum along the direction of linear dilaton growth measured in string units. μ again serves as a characteristic scale indicating when new physics sets in and perturbation theory breaks down. In this model the new physics is the possibility of fermions going over the potential

¹¹ These C_l have been computed in the dual heterotic picture in the beautiful paper [30]. As mentioned in [30] terms like F^{2l} would naturally occur in studying the perturbation expansion of the theory in a uniform RR field strength. Perhaps the nonperturbative effects associated with the $c = 1$ model are, in this context, to be interpreted as pair production of light black holes in a background field. The identification of μ with the black hole mass discussed later in this section supports this. This idea was developed in a discussion with Mike Douglas and Andy Strominger and is also implicit in [30].

¹² This observation was developed in discussions with Hiroshi Ooguri and Cumrun Vafa. We hope to report on further developments in the not too distant future.

barrier of height μ . There is a simple picture for this kind of momentum dependence. The higher the momentum down the tube, the deeper the particle can penetrate into the tachyon condensate “wall” and hence experience a stronger coupling. Perhaps a related high momentum behavior occurs in conifold amplitudes. The new physics signaled by this behavior would presumably have to do with black hole creation, as black holes have mass μ in string units. Such behavior would signal the breakdown of string perturbation theory and perhaps its associated soft high momentum behavior.

6. Some Puzzles

There are many puzzles posed by the possible existence of this new scale. In this section we will discuss some of them.

We have argued that interpreting (2.2) as the result of a one loop vacuum polarization graph, where a low momentum RR gauge field excites a virtual charged black hole pair, requires that the loop momentum of the black hole run up to $\sim m_s/g_s$. But the RR gauge field is a conventional perturbative string state which we might have thought would have “size” $1/m_s$. How can it couple in a pointlike way to the charged black hole all the way up to momenta of order $\sim m_s/g_s$?¹³ We might be able to think of the structure of the RR vector state as coming from the exchange of additional neutral perturbative string states in the diagram. We don’t really know how to work with such diagrams, since we don’t know the coupling of perturbative string states to light black holes, and the coupling of the black holes to RR vectors is order one. Nonetheless we might be able to attribute the absence of string scale effects in τ_{RR} to a cancellation analogous to that responsible for the $\mathcal{N} = 2$ nonrenormalization theorems that prevent neutral hypermultiplets from contributing here. Such exchanges would affect other quantities and suggest a string scale “halo” surrounding these solitons that special quantities like τ_{RR} don’t see. The ultraviolet cutoff at m_s/g_s would correspond to a much smaller “charge radius” of the soliton.

The $\exp(-1/g_s)$ nonperturbative effects expected at weak coupling [46] are another indication of such a “halo.” World line loops of RR solitons were proposed as a mechanism for such effects in [12]. In order for the action of such a loop to be $\sim 1/g_s$ the loop’s size must be $\sim 1/m_s$. If there is no suppression of smaller loops then the contribution of these solitons will only be power law suppressed by inverse powers of their mass.¹⁴ This

¹³ This puzzle arose in a discussion with Lenny Susskind.

¹⁴ These points were made by Lenny Susskind.

suggests a “size” for the solitons $\sim 1/m_s$. Supersymmetry nonrenormalization theorems do not prohibit perturbative string state exchanges from affecting, e.g., the coefficient c_4 of F^4 where F is the RR field strength. The general arguments of [46] would suggest $(2g)!$ large order growth in the perturbation series of c_4 since it is not protected by supersymmetry. We would then expect $\exp(-1/g_s)$ corrections to c_4 . Since the string state exchanges do not cancel they could stabilize the minimal loop of RR solitons coupling to such a quantity at a length $\sim 1/m_s$, giving the requisite action. An important test of such a picture would be to isolate the nonperturbative effects contributing to such a quantity from the light solitons of mass $|Z|m_s/g_s$. These would be of order $\exp(-Z/g_s)$ assuming a minimum loop size $\sim 1/m_s$. The corresponding large order perturbative behavior of a four vector scattering amplitude at low momentum p and $Z \rightarrow 0$ would look schematically like

$$S(p) \sim p^4 \sum_l (g_s^2/Z^2)^l (2l!) . \quad (6.1)$$

Note the contrast with (5.2). Understanding the full momentum dependence of the double scaling limit of such scattering amplitudes will be instructive.

If, at weak coupling, the solitons should be thought of as having a “halo” $\sim 1/m_s$ perhaps all direct effects of their existence will be exponentially suppressed.

In the type II language the conifold phenomenon is in some sense a weak coupling one, although some perturbative states are certainly strongly coupled when $m_{bh} \rightarrow 0$ even though g_s can be arbitrarily small. Nonetheless we might expect to get some insight into the occurrence of the m_s/g_s scale by examining the classical solution for the black hole.¹⁵

Strominger has argued that the appropriate classical configuration is the threebrane of the 10D IIB theory [50] wrapped around the shrinking three-cycle of the CY. The threebrane has a “throat” whose width defines a characteristic size. We might expect this size to be string scale, but in fact it is not. The threebrane only carries RR charge (described by the integral of the self-dual five-form) and so the peculiarities of RR charge quantization come into play. The dilaton can be completely eliminated from the Lagrangian by going to Einstein frame and rescaling the RR fields so they are quantized in unit strength. The size must be, then, the only scale remaining, which is the Planck scale. Note that this is the ten dimensional Planck scale $m_P^{(10)} \sim m_s/g_s^{1/4}$, *not* the four dimensional Planck scale $m_P^{(4)} \sim m_s/g_s$. Other solitons that do carry NS charge, for example the symmetric

¹⁵ I thank Andy Strominger for teaching me most of what follows.

fivebrane used to construct the heterotic soliton [13][14], have throat sizes that are string scale.

What apparently is missing in this classical description is the analog of the W bosons discussed in section 4 that seem to be providing the physical cutoff for RR charge fluctuations.

It will be important to look at this scale issue in various dimensions, (e.g., for the massless black holes that appear at the degenerations of K3 in 6D type II string theory), to discriminate between m_P and m_s/g_s which happen to coincide in four dimensions. The mass of RR solitons is always $\sim m_s/g_s$, independent of dimension [12].

As an aside, let us point out that the peculiar quantization of RR charge gives an intuitive explanation for the unusual lightness of RR solitons as compared to conventional solitons whose mass is $\sim m_s/g_s^2$. The low energy field theory measure with fields defined in units natural to string theory looks schematically like $\exp(-S/g_s^2)$ where S is the action. Now typical classical field configurations corresponding to solitons have field magnitudes of order unity and so $S \sim 1$. The typical soliton mass (derived from the action for a world line) is then $\sim m_s/g_s^2$. But the RR quantization condition in string units requires that the RR field strength $F \sim g_s$ and so the RR field is much weaker than unity. This allows the action to be ¹⁶ $\sim g_s$ and the soliton mass to be $\sim m_s/g_s$.¹⁷

Another important puzzle concerns the limit $g_s \rightarrow 0$, Z small but fixed. In this limit string perturbation theory is accurate and so there must be a conventional conformal field theory explanation for the logarithm in (2.2). As discussed in [47][17][32] the CY conformal field theory produces many perturbative string states going to zero mass as $Z \rightarrow 0$ which have couplings $\sim g_s/Z$. These states, in some sense, account for the logarithm. As $Z \rightarrow 0$ with g_s fixed the black hole mass drops below m_s , the light perturbative states become strongly coupled and the description in terms of light black holes becomes the only simple one available. But somehow these two descriptions must be connected. The light perturbative string states should be related to black holes. These states are neutral (as are all perturbative states) so one might guess that they are some kind of black hole-anti black hole composites. They would remain light as the black holes became heavy because

¹⁶ In fact, the bulk action is zero and the mass is given by a surface term. I thank Tom Banks for a discussion about this.

¹⁷ This is reminiscent of the one eigenvalue instanton mechanism that produces $\exp(-1/g_s)$ effects in matrix models.

the black holes interact strongly with each other. The Sine-Gordon model might provide an analogy. There the solitons form bound states called breathers. As $\hbar \rightarrow 0$ the low lying breathers remain light even though the solitons become infinitely heavy. Motivated by the connection mentioned in section 5, we might think of the $c = 1$ matrix model as another analog. The very light “tachyons” are particle-hole excitations of the fermi surface. There are other excitations of energy $\mu (= m_{\text{bh}})$ corresponding to particles flying over the barrier. As $\mu \rightarrow \infty$ the “tachyons” remain light. To complete the story we would have to understand how the one loop diagram evolves into some kind of bound state exchange as $g_s \rightarrow 0$.

Another question raised by the existence of a short scale concerns finiteness. We are used to thinking that the softness of string theory at momenta higher than m_s is crucial to the theory’s good ultraviolet behavior. But if there is another much shorter dynamical length scale in the problem how do we know that the theory still stays well behaved?

Now we know that even close to the conifold limit, string perturbation theory is finite order by order in g_s^2 . So for example we know that the log is cut off. To what extent does this assure us that the full theory is uv finite?

Finally we can ask about this short length scale far away from the conifold limit, say $Z \sim 1$. At weak coupling there is nothing unusual about the spectrum—the light states discussed above have moved up to string scale. Do they still represent black hole bound states? Are all direct effects of black holes at low momenta suppressed by $\exp(-1/g_s)$?

Of course the biggest question raised by the existence of another short length scale is what lesson it is trying to tell us about the correct formulation of string theory.

7. Concluding Remarks

We have seen that several puzzles – that τ_{RR} behaves as $\log(Z)$ and not $\log(Z/g_s)$, that τ_{RR} is essentially independent of R after compactification, and aspects of the dual mapping of the conifold singularity onto the Seiberg-Witten massless monopole point—can be understood by positing the existence of another dynamical scale $m_{\text{uv}} \sim m_s/g_s$. On the other hand the existence of such a scale raises several puzzles of its own.

To better understand these issues it will be important to assemble more evidence for this scale, especially in dimensions other than four where $m_P \neq m_s/g_s$. Hopefully we can find evidence of a more direct nature whose form is not so strongly constrained by supersymmetry.

As we search, though, we must bear in mind the very real possibility that there is another explanation for these puzzles that does not invoke such a surprising new element.

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